

RIMS Workshop

Analytic, geometric and p -adic aspects of automorphic forms and L -functions

Main organizer : Shunsuke Yamana (Osaka City University)

Sub-organizer : Takuya Yamauchi (Tohoku University)

Date : January 20 (Mon.) - 24 (Fri.), 2020.

Venue : Room 420, Research Institute of Mathematical Science, Kyoto University.

Jan. 20 (Mon.)

12:30 – 13:30 **Kazuki Morimoto** (Kobe University)

On Gan-Gross-Prasad conjecture for $(U(2n), U(1))$ and $(SO(5), SO(2))$.

Abstract: Gan, Gross and Prasad conjectured a relationship between non-vanishing of certain periods and non-vanishing of central values of certain tensor product L -functions. In this talk, I will show this conjecture in the case of $(U(2n), U(1))$ for tempered cuspidal automorphic representations. Moreover, I will show a similar result for $(SO(5), SO(2))$ using the case of $(U(4), U(1))$. This is a joint work with Masaaki Furusawa.

13:40 – 14:40 **Bingchen Lin** (Sichuan University, China)

Archimedean Non-Vanishing and Cohomological Test Vector

Abstract: The standard L -functions of GL_{2n} expressed in terms of the Friedberg-Jacquet global zeta integrals have better structure for arithmetic applications, due to the relation of the linear periods with the modular symbols. In this talk, we will focus on the archimedean local integrals of Friedberg-Jacquet. We will give the explicit construction of uniform cohomological test vector v and establish the non-vanishing property for the archimedean local Friedberg-Jacquet integral when evaluating at v .

14:50 – 15:50 **Zheng Liu** (University of California, Santa Barbara)

Doubling archimedean zeta integrals for symplectic and unitary groups

Abstract: In order to verify the compatibility between the conjecture of Coates–Perrin-Riou and the interpolation results of the p -adic L -functions constructed by using the doubling method, a doubling archimedean zeta integral needs to be calculated for holomorphic discrete series. When the holomorphic discrete series is of scalar weight, it has been done by Bocherer–Schmidt and Shimura. I will explain a way to compute this archimedean zeta integral for general vector weights by using the theory of theta correspondence.

16:00 – 17:00 **Tadashi Miyazaki** (Kitasato University):

Recurrence relations for archimedean Rankin-Selberg integrals

Abstract: Let n and m be positive integers such that $n - m = 0, 1$. For pairs of spherical principal series representations, Eric Stade calculate archimedean Rankin-Selberg integrals for $GL(n) \times GL(m)$ explicitly, using recurrence relations for them. In this talk, we generalize his result to ‘compatible’ pairs of (non-spherical) principal series representations. Moreover, we introduce an application to the non-vanishing hypothesis for compatible pairs of cohomological representations at the complex place. This is a joint work with Taku Ishii.

Jan. 21 (Tue.)

9:30 – 10:30 **Kazuaki Tajima** (National Institute of Technology, Sendai College):

On the GIT stratification of prehomogeneous vector spaces.

Abstract: We have established a combinatorial method to determine a stratification (GIT stratification) based on geometric invariant theory. We applied this method to certain prehomogeneous vector spaces using computer computations and determined their GIT stratifications. The result has been known if the ground field is \mathbb{C} . This method enables us to determine the stratification rationally over any perfect field. This is joint work with Akihiko Yukie.

10:45 – 11:45 **Hiro-aki Narita** (Waseda University)

Fourier-Jacobi expansion of non-holomorphic real analytic cusp forms on $Sp(2, \mathbb{R})$.

Abstract: The Fourier-Jacobi expansion has been a powerful tool to study holomorphic Siegel modular forms. Though we naturally expect a similar thing for non-holomorphic cases it is quite difficult to understand it in detail. The aim of this talk is to develop a theory of Fourier-Jacobi expansion for non-holomorphic Siegel cusp forms on the Siegel upper half space of degree two, or on $Sp(2, \mathbb{R})$. Toward this there was the work by Miki Hirano on “Fourier-Jacobi type spherical functions on $Sp(2, \mathbb{R})$ ”. Though this explicitly describes the special functions necessary to provide the Fourier-Jacobi expansion for non-holomorphic cases, there have been no trial to apply Hirano’s result to write down such Fourier-Jacobi expansion. The key idea to carry out this is a representation theoretic generalization of the Eichler-Zagier correspondence between the “intertwining operator valued” cusp forms of half-integral weight and Jacobi cusp forms, the latter of which are defined in a representation theoretic manner. This also includes a representation theoretic generalization of the theta decompositions of Jacobi forms. The Jacobi forms involved in this theory form a general class of Jacobi forms including Maass-Jacobi forms. This talk will mainly take up the case of cusp forms generating large discrete series representation though the theory is applicable for more general cases.

13:30 – 14:30 **Seiji Kuga** (Kyushu University)

On linear relations for special L -values over certain totally real number fields.

Abstract: Hiraga and Ikeda gave a generalization of the Kohnen plus space for Hilbert modular forms. Ren-He Su constructed an Eisenstein series belonging to the generalized Kohnen plus space of parallel half integral weight and showed that their Fourier coefficients can be written as a linear combination of special L -values. By using these results, he gave linear relations between special L -values over certain real quadratic fields and some arithmetic functions. In this talk, we explain a generalized linear relations for special L -values over certain totally real number fields.

14:45 – 15:45 **Yota Maeda** (Kyoto University)

On the modularity of special cycles on orthogonal Shimura varieties.

Abstract: In 1990, Kudla-Millson proved that the generating series of special cycles on a Shimura variety is a Hilbert-Siegel modular form in the cohomology group. These work is the beginning of “Kudla’s program”. Recently, Yuan-Zhang-Zhang studied the generating series associated with special cycles on a certain orthogonal Shimura variety and showed its modularity in the Chow group. In this talk, assuming the Beilinson-Bloch conjecture, we

generalize their results to more general orthogonal Shimura varieties.

16:00 – 17:00 **Hang Xue** (University of Arizona)

Towards a factorization of linear periods.

Abstract: I will explain how to relate local root numbers to the existence of linear models of representation of $GL(2n)$. I will also explain how to make use of this to prove conjectures of Sakellaridis and Venkatesh on the Plancherel formula for $GL(n, E) \backslash GL(2n, F)$ and on the canonical factorization of linear periods.

Jan. 22 (Wed.)

9:20 – 9:30 On the RIMS workshop next academic year.

9:30 – 10:30 **Tadashi Ochiai** (Osaka University)

Endoscopic congruences and adjoint L -values for $GSp(4)$.

Abstract: In his paper published in 1981, Hida proved the existence of a mod p congruence between two elliptic normalized eigen cuspforms f and g of the same weight and level when the prime p divides the special value at 1 of the adjoint L -function of the form f . After reviewing this classical result, we explain an extension of the existence of congruences between a fixed endoscopic cuspidal automorphic representation of $GSp(4)$ of square-free conductor and non-endoscopic cuspidal automorphic representations of the same level and weight modulo certain prime factors of the value at 1 of the adjoint L -function normalized by a suitable period. This is a joint work with Francesco Lemma.

10:45 – 11:45 **Ming-Lun Hsieh** (National Taiwan University)

Restriction of Eisenstein series and the factorization of p -adic twisted triple product L -series

Abstract: In a recent work, Darmon, Pozzi and Vonk consider a special weight-variable p -adic twisted triple product L -series obtained from the pull-back of certain one variable p -adic family of Eisenstein series on a real quadratic field and show that the derivative this p -adic L -series at weight one can be expressed in terms of p -adic logarithms of Stark-Heegner points and central L -values of modular forms of weight two. We will talk about a generalization of their work to a two-variable setting and explain the factorization formula of this twisted p -adic triple product L -series into the product of the square root p -adic L -series associated with a Hida family over a real quadratic field and a two-variable Mazur-Kitagawa p -adic L -function. This is a joint work with Shunsuke Yamana.

13:30 – 14:30 **Kenichi Namikawa** (Kyushu University)

A construction of p -adic Asai L -functions and related topics.

Abstract: In this talk, we consider so-called Asai representations, which are a twisted tensor product of automorphic representations of $GL(2)$ over CM fields and we discuss on a construction of p -adic analogue of L -functions attached to it. If the base field is imaginary quadratic, it can be found a work by Loeffler-Williams. We introduce a generalization of their construction.

14:45 – 15:45 **Ameya Pitale** (University of Oklahoma)

Maass forms on $GL(2)$ over division quaternion algebras of discriminant p .

Abstract: In this talk, we will present a construction of Maass forms F on 5 dimensional hyperbolic space via lifting from Maass forms f of level p . These lifts correspond to cuspidal

automorphic representations on $GL(2)$ over a division quaternion algebra with discriminant p . The lifts F are given explicitly by presenting a formula for their Fourier coefficients in terms of those of f . The automorphy is proven via Borcherds explicit theta lift. The corresponding cuspidal representation is a CAP representation and violates the Generalized Ramanujan conjecture. This is joint work with Hiro-aki Narita and Siddhesh Wagh.

16:00 – 17:00 **Kengo Fukunaga** (Osaka University)

Triple product p -adic L -function attached to p -adic families of modular forms.

Abstract: Ming-Lun Hsieh constructed three-variable p -adic triple product L -functions attached to triples (F, G, H) of primitive Hida families in his paper “Hida families and p -adic triple product L -functions” (AJM, to appear). I generalized his result in the unbalanced case and constructed a three-variable p -adic triple product L -function attached to a primitive Hida family F and two more general p -adic families of modular forms G, H . We can take more general families for G, H such as Coleman families or CM-families.

18:00 – Dinner Party.

Jan. 23 (Thu.)

9:30 – 10:30 **Ramla Abdellatif** (Université de Picardie Jules Verne)

Restriction of p -modular representations of p -adic groups to minimal parabolic subgroups.

Abstract: Given a prime integer p , a non-archimedean local field F of residual characteristic p and a standard Borel subgroup P of $GL_2(F)$, Paškūnas proved that the restriction to P of (irreducible) smooth representations of $GL_2(F)$ over $\overline{\mathbb{F}}_p$ encodes a lot of information about the full representation of $GL_2(F)$ and that it leads to useful statement about p -adic representations of $GL_2(F)$. Nevertheless, the methods used at that time by Paškūnas heavily relied on the understanding of the action of certain spherical Hecke operator and on some combinatorics specific to the $GL_2(F)$ case. This method can be transposed case by case to for some other quasi-split groups of rank 1, but this is not very satisfying as such.

This talk will report on a joint work with J. Hauseux. Using Emerton’s ordinary parts functor, we get a more uniform context which shed a new light on Paškūnas’ results and allows us to generalize very naturally these results for arbitrary rank 1 groups. In particular, we prove that for such groups, the restriction of supersingular representations to a minimal parabolic subgroup is always irreducible.

10:45 – 11:45 **Vincent Sécherre** (Université de Versailles Saint-Quentin)

Congruence properties of endo-classes and the local Jacquet-Langlands correspondence.

Abstract: Let F be a non-Archimedean local field of residue characteristic p , let G be an inner form of $GL(n, F)$ and π be an irreducible smooth complex representation of G . Associated with π , there is a type theoretic invariant called its endo-class, which is a class of certain characters of certain pro- p subgroups of G occurring in π . When G is split, this invariant is related, through the local Langlands correspondence, to the restriction of the Langlands parameter of π to the wild inertia subgroup. We prove that the fact that two irreducible representations of G have the same endo-class can be formulated in terms of congruence properties mod a set of prime numbers different from p . As a consequence, one can prove that the local Jacquet-Langlands correspondence preserves endo-classes, and it can be explicitly described, up to inertia, in terms of Bushnell-Kutzko types. (Joint work

with Shaun Stevens.)

13:30 – 14:30 **Yuki Yamamoto** (University of Tokyo)

On the types for supercuspidal representations of inner forms of GL_N .

Abstract: Let G be the multiplicative group of a central simple algebra over a non-archimedean local field. To classify smooth representations of G , the representations called types are useful. In this talk, I explain types and discuss the existence and uniqueness of types for irreducible supercuspidal representations π of G . In particular, I will show that $[G, \pi]_G$ -types defined over some maximal compact open subgroup of G are unique up to G -conjugation under some unramifiedness assumption on π .

14:45 – 15:45 **Winfried Kohnen** (Heidelberg University)

On Fourier coefficients of modular forms of half-integral weight

Abstract: We will introduce and study a certain Dirichlet-type L -function attached to a modular form of half-integral weight. We will give applications for the squarefree Fourier coefficients. This is joint work with S. Gun (2019/20).

16:00 – 17:00 **Keiichi Gunji** (Chiba Institute of Technology)

On the Fourier coefficients of Siegel Eisenstein series of odd level and the genus theta series.

Abstract: Siegel Eisenstein series are one of the most important and typical example of the Siegel modular forms, whose Fourier coefficients have studied by many mathematicians. The speaker have calculated the Fourier coefficients of Siegel Eisenstein series with levels and characters, in the case of degree 2 or 3. In this talk, we will explain that by using genus theta series, we can get an explicit formula of the Fourier coefficients of Eisenstein series of general degree with levels and trivial or quadratic characters.

Jan. 24 (Fri.)

9:10 – 10:10 **Biplab Paul** (Kyushu University)

Growth of Petersson inner products of Fourier-Jacobi coefficients of Siegel cusp forms.

Abstract: The Fourier-Jacobi coefficients of Siegel cusp forms have been studied for a long time. In this talk, we shall discuss certain growth properties of Petersson inner products of Fourier-Jacobi coefficients of Siegel cusp forms. In particular, we show that the Ramanujan-Petersson conjecture for Petersson norm of Fourier-Jacobi coefficients of Siegel cusp forms is true on average for arbitrary degree. We also show that this conjecture is true for Ikeda lifts.

10:20 – 11:20 **Akihiko Yukie** (Kyoto University):

On the density theorem related to the space of non-split tri-Hermitian forms

Abstract: Let k be a fixed cubic field which is unramified at 3. For a quadratic field F , let $L = F \cdot k$. In this talk we discuss the density of $h_L R_L / h_F R_F$ (the relative hR). If k is non-normal, we proved that $\sum_{0 < |\Delta_F| < X} h_L R_L / h_F R_F$ is asymptotically CX^2 where C is a constant expressed as an Euler product. If k is cyclic, we still have a density theorem with an additional factor related to the global-local behavior of the torus $\text{Ker}(L^\times \rightarrow F^\times)$. Explicit trace formula of $SL_3(\mathbb{Z})$ and its application.

11:30 – 12:30 **Tomoyoshi Ibukiyama** (Osaka University)

Pullback formulas and applications.

Abstract: We give a pullback formula of Jacobi Eisenstein series of general degree under operation of automorphic differential operators. As applications of the formula, we explain (1) Several kinds of rationality of Fourier coefficients of Jacobi forms. (2) Algebraicity of critical values of Sugano-Murase's L functions of Jacobi forms. (3) Algebraicity of Spinor L function of Siegel modular forms of degree two under conjectural correspondence of Shimura type between integral and half-integral weight. (4) Half-integral version of the Harder conjecture.